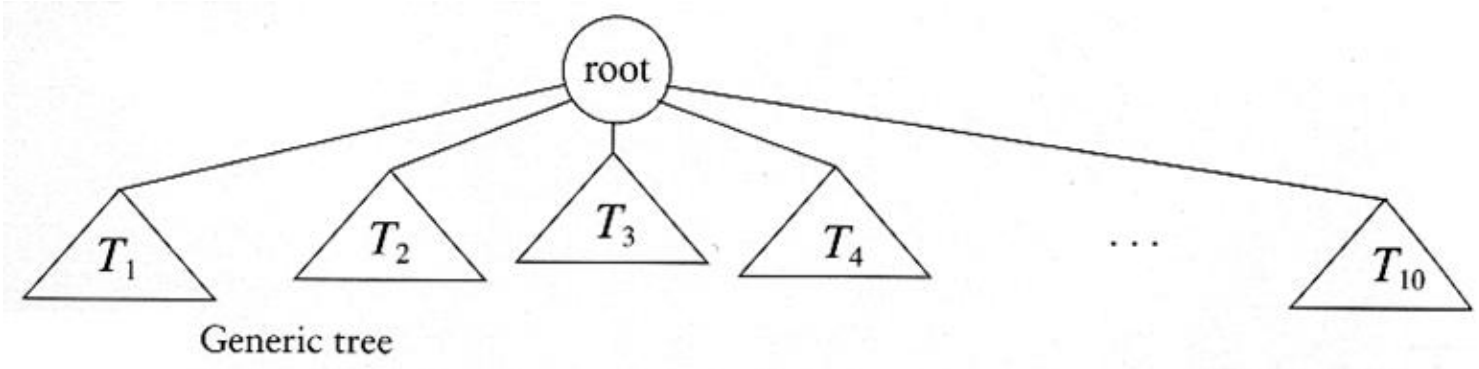
Binary Trees,
Binary Search Trees

Trees

- Linear access time of linked lists is prohibitive
  - Does there exist any simple data structure for which the running time of most operations (search, insert, delete) is O(log N)?

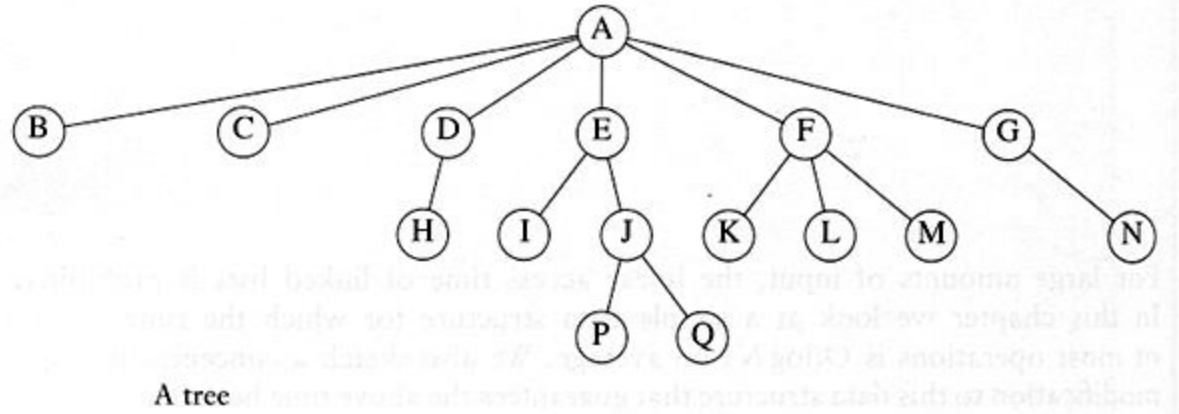
Trees

- A tree is a collection of nodes
  - The collection can be empty
  - (recursive definition) If not empty, a tree consists of a distinguished node r (the *root*), and zero or more nonempty *subtrees* T_1, T_2, ..., T_k, each of whose roots are connected by a directed *edge*



Generic tree

Some Terminologies



A tree

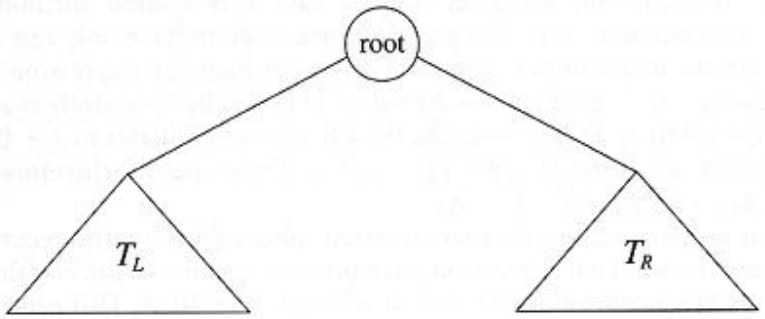
- *Child* and *parent*
  - Every node except the root has one parent
  - A node can have an arbitrary number of children
- *Leaves*
  - Nodes with no children
- *Sibling*
  - nodes with same parent

Some Terminologies

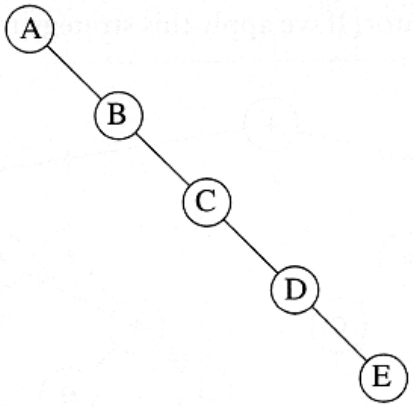
- *Path*
- *Length*
  - number of edges on the path
- *Depth* of a node
  - length of the unique path from the root to that node
  - The depth of a tree is equal to the depth of the deepest leaf
- *Height* of a node
  - length of the longest path from that node to a leaf
  - all leaves are at height 0
  - The height of a tree is equal to the height of the root
- *Ancestor* and *descendant*
  - *Proper ancestor* and *proper descendant*

Binary Trees

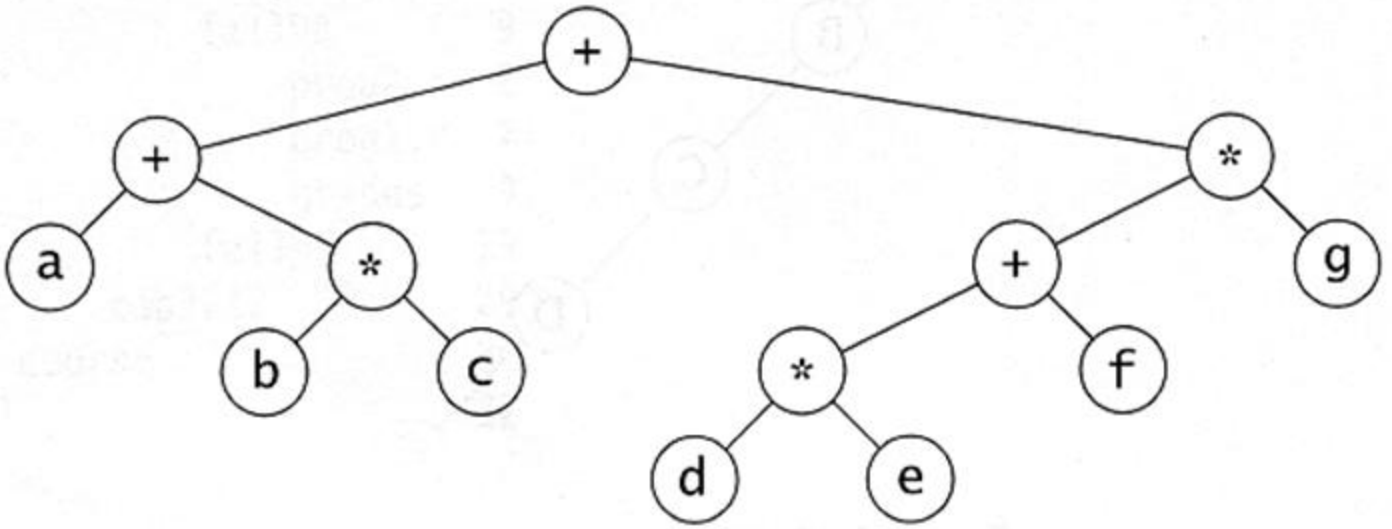
- A tree in which no node can have more than two children



- The depth of an “average” binary tree is considerably smaller than N, eventhough in the worst case, the depth can be as large as N – 1.



Example: Expression Trees



Expression tree for (a + b * c) + ((d * e + f) * g)

- Leaves are operands (constants or variables)
- The other nodes (internal nodes) contain operators
- Will not be a binary tree if some operators are not binary

Tree traversal

- Used to print out the data in a tree in a certain order
- Pre-order traversal
  - Print the data at the root
  - Recursively print out all data in the left subtree
  - Recursively print out all data in the right subtree

Preorder, Postorder and Inorder

- Preorder traversal
  - node, left, right
  - prefix expression
    - ++a*bc*+*defg

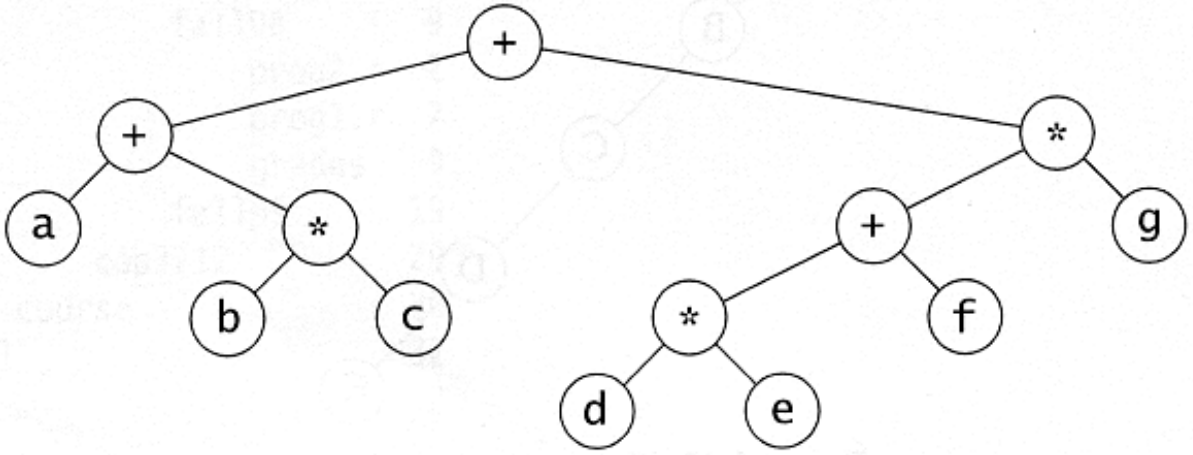


Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

Preorder, Postorder and Inorder

- Postorder traversal
  - left, right, node
  - postfix expression
    - abc*+de*f+g*+
- Inorder traversal
  - left, node, right.
  - infix expression
    - a+b*c+d*e+f*g

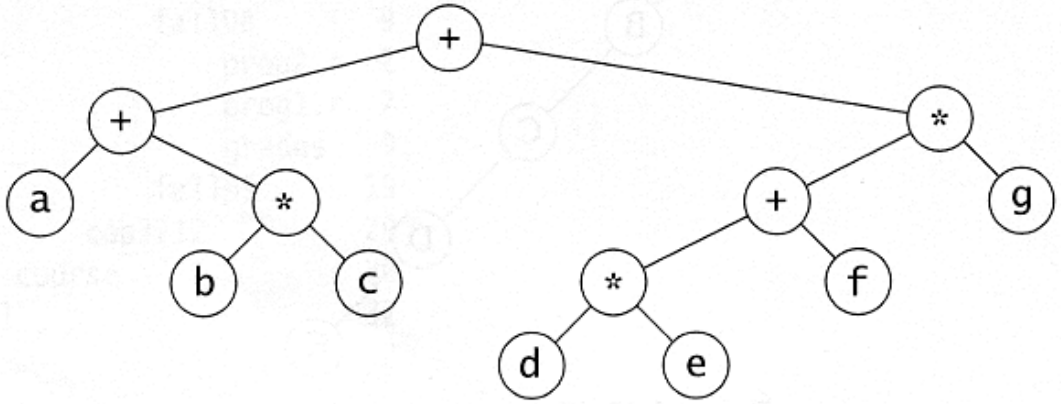


Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

- Preorder

```
/usr
    mark
        book
            ch1.r
            ch2.r
            ch3.r
        course
            cop3530
                fall98
                    syl.r
                spr99
                    syl.r
                sum99
                    syl.r
        junk
    alex
        junk
    bill
        work
        course
            cop3212
                fall98
                    grades
                    prog1.r
                    prog2.r
                fall99
                    prog2.r
                    prog1.r
                    grades
```

- Postorder

```
                ch1.r           3
                ch2.r           2
                ch3.r           4
            book               10
                        syl.r   1
                    fall98      2
                        syl.r   5
                    spr99       6
                        syl.r   2
                    sum99       3
                cop3530        12
            course             13
            junk                6
        mark                   30
            junk                8
        alex                    9
            work                1
                        grades  3
                        prog1.r 4
                        prog2.r 1
                    fall98      9
                        prog2.r 2
                        prog1.r 7
                        grades  9
                    fall99     19
                cop3212        29
            course             30
        bill                   32
/usr                           72
```

Preorder, Postorder and Inorder

```
Algorithm Preorder(x)
Input: x is the root of a subtree.
1.    if x ≠ NULL
2.       then output key(x);
3.            Preorder(left(x));
4.            Preorder(right(x));
```

```
Algorithm Postorder(x)
Input: x is the root of a subtree.
1.    if x ≠ NULL
2.       then Postorder(left(x));
3.            Postorder(right(x));
4.            output key(x);
```

```
Algorithm Inorder(x)
Input: x is the root of a subtree.
1.    if x ≠ NULL
2.       then Inorder(left(x));
3.            output key(x);
4.            Inorder(right(x));
```

Binary Trees

- Possible operations on the Binary Tree ADT
  - parent
  - left_child, right_child
  - sibling
  - root, etc
- Implementation
  - Because a binary tree has at most two children, we can keep direct pointers to them

```
struct BinaryNode
{
    Object      element;      // The data in the node
    BinaryNode *left;         // Left child
    BinaryNode *right;        // Right child
};
```

compare: Implementation of a general tree

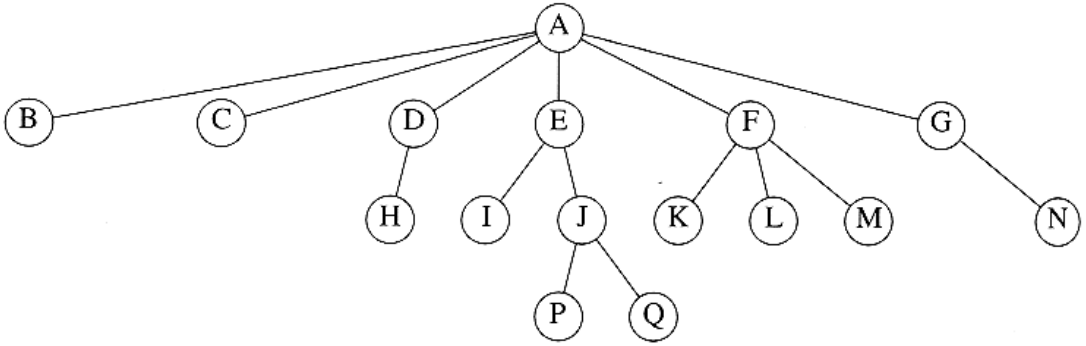


Figure 4.2 A tree

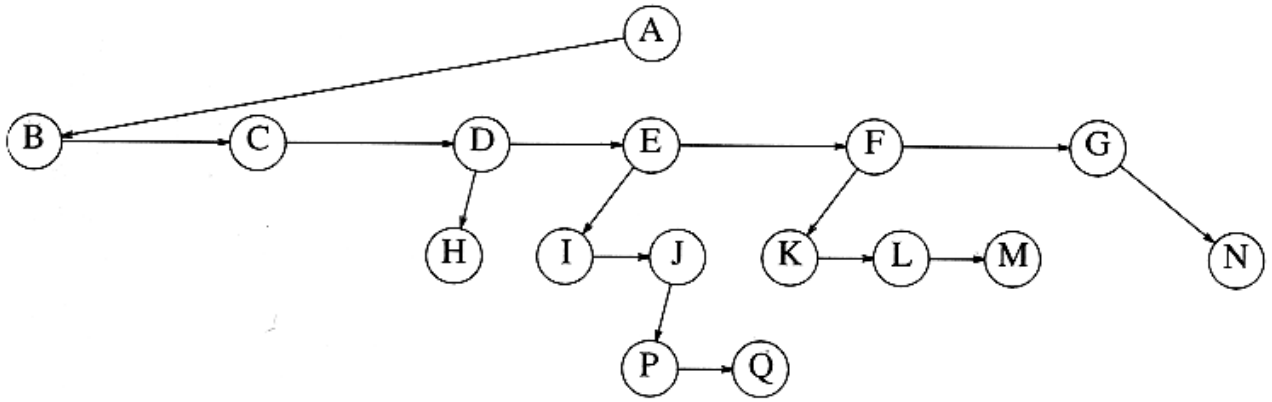


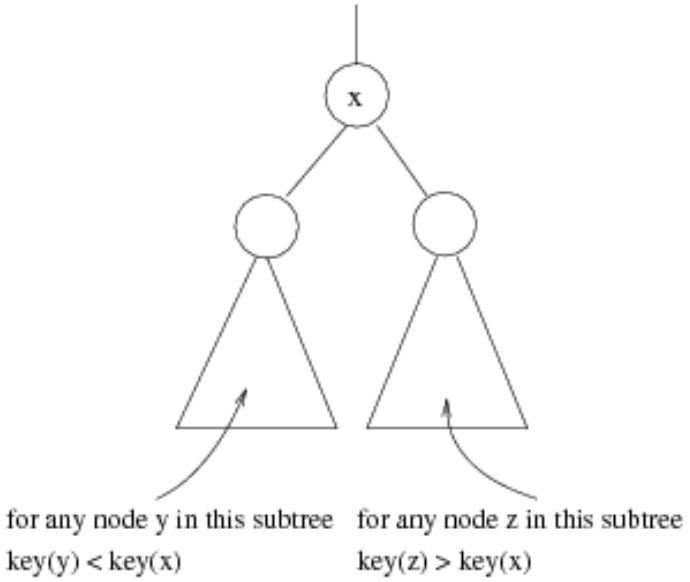
Figure 4.4 First child/next sibling representation of the tree shown in Figure 4.2

Binary Search Trees

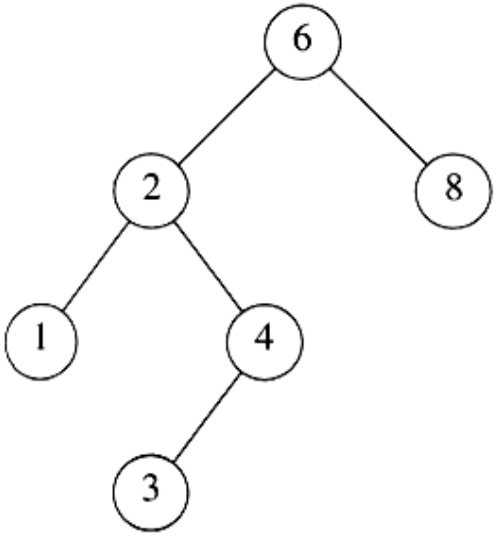
- Stores keys in the nodes in a way so that searching, insertion and deletion can be done efficiently.

Binary search tree property

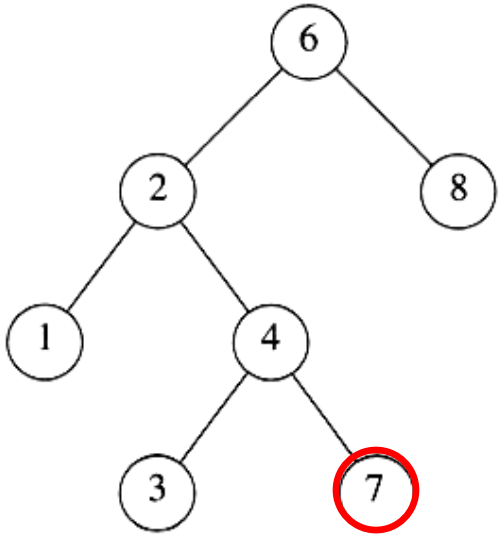
- For every node X, all the keys in its left subtree are smaller than the key value in X, and all the keys in its right subtree are larger than the key value in X



Binary Search Trees



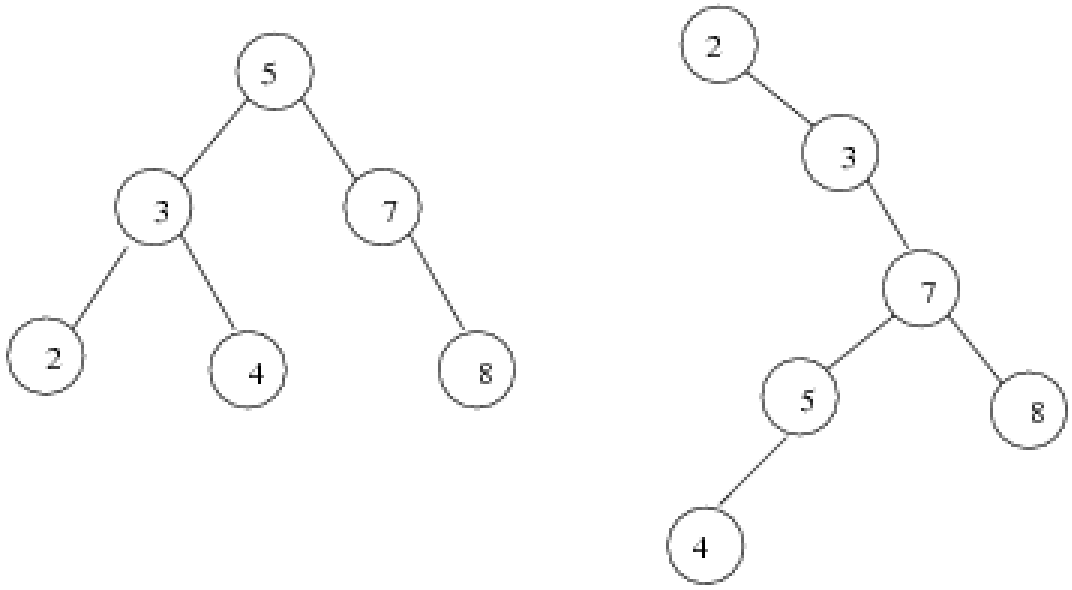
A binary search tree



Not a binary search tree

Binary search trees

Two binary search trees representing the same set:



- Average depth of a node is O(log N); maximum depth of a node is O(N)

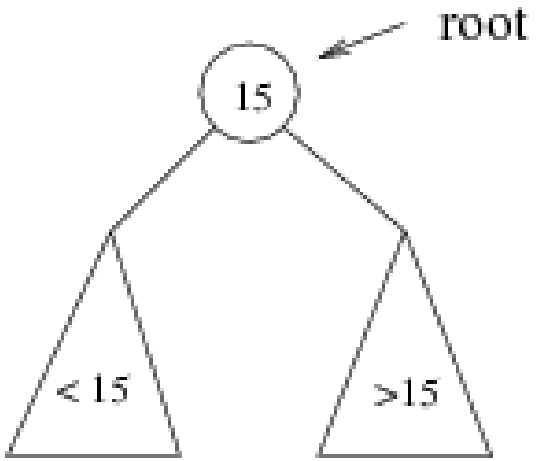
Implementation

```
template <class Comparable>
class BinarySearchTree;

template <class Comparable>
class BinaryNode
{
    Comparable  element;
    BinaryNode *left;
    BinaryNode *right;

    BinaryNode( const Comparable & theElement, BinaryNode *lt,
                                               BinaryNode *rt )
      : element( theElement ), left( lt ), right( rt ) { }
    friend class BinarySearchTree<Comparable>;
};
```

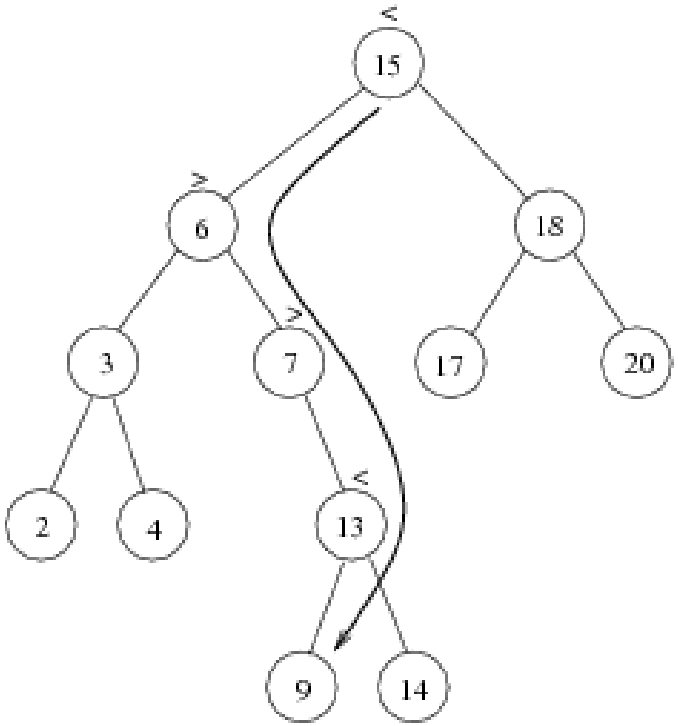
Figure 4.16 The BinaryNode class

Searching BST

- If we are searching for 15, then we are done.
- If we are searching for a key < 15, then we should search in the left subtree.
- If we are searching for a key > 15, then we should search in the right subtree.



Example: Search for 9 ...



Search for 9:

1. compare 9:15(the root), go to left subtree;
2. compare 9:6, go to right subtree;
3. compare 9:7, go to right subtree;
4. compare 9:13, go to left subtree;
5. compare 9:9, found it!

Searching (Find)

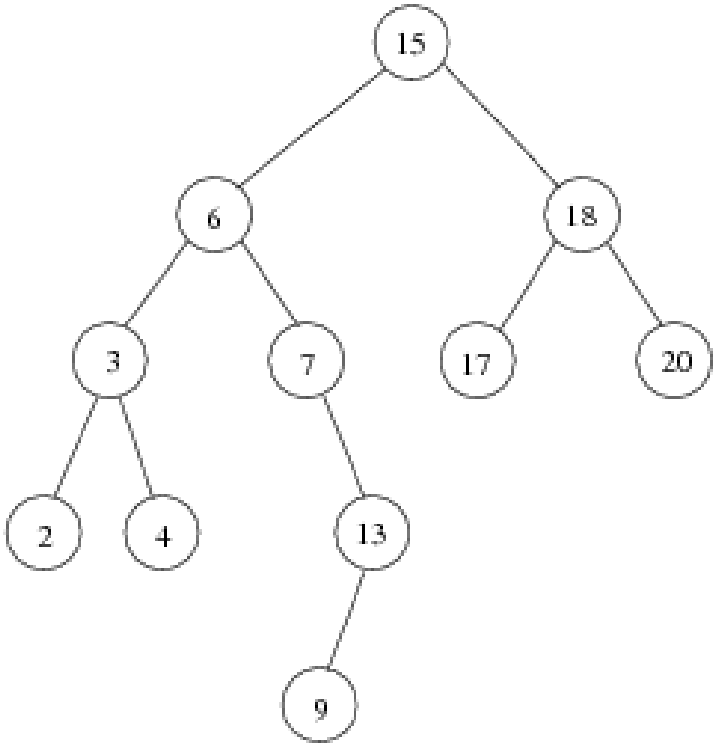
- Find X: return a pointer to the node that has key X, or NULL if there is no such node

```
template <class Comparable>
BinaryNode<Comparable> *
BinarySearchTree<Comparable>::
find( const Comparable & x, BinaryNode<Comparable> *t ) const
{
    if( t == NULL )
        return NULL;
    else if( x < t->element )
        return find( x, t->left );
    else if( t->element < x )
        return find( x, t->right );
    else
        return t;    // Match
}
```

- Time complexity
  - O(height of the tree)

Inorder traversal of BST

- Print out all the keys in sorted order



Inorder: 2, 3, 4, 6, 7, 9, 13, 15, 17, 18, 20

findMin/ findMax

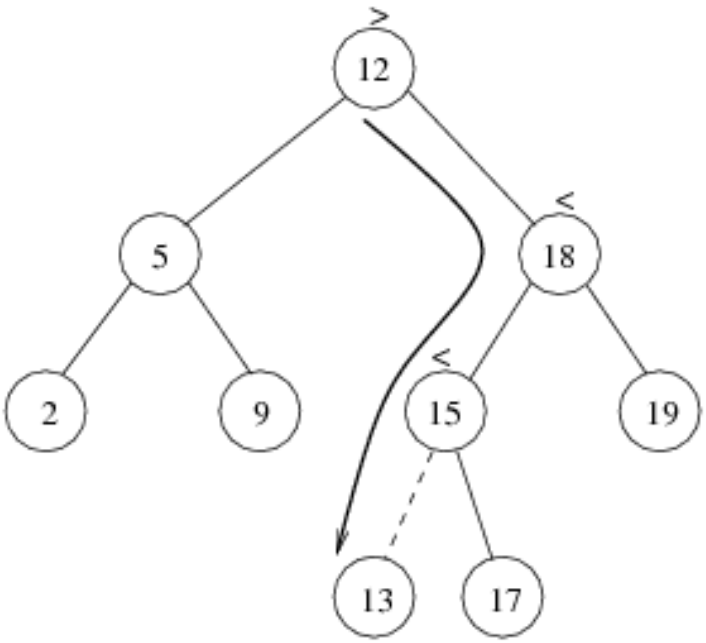
- Return the node containing the smallest element in the tree
- Start at the root and go left as long as there is a left child. The stopping point is the smallest element

```
template <class Comparable>
BinaryNode<Comparable> *
BinarySearchTree<Comparable>::findMin( BinaryNode<Comparable> *t ) const
{
    if( t == NULL )
        return NULL;
    if( t->left == NULL )
        return t;
    return findMin( t->left );
}
```

- Time complexity = O(height of the tree)

insert

- Proceed down the tree as you would with a find
- If X is found, do nothing (or update something)
- Otherwise, insert X at the last spot on the path traversed



- Time complexity = O(height of the tree)

delete

- When we delete a node, we need to consider how we take care of the children of the deleted node.
  - This has to be done such that the property of the search tree is maintained.

delete

Three cases:

(1) the node is a leaf

- Delete it immediately

(2) the node has one child

- Adjust a pointer from the parent to bypass that node

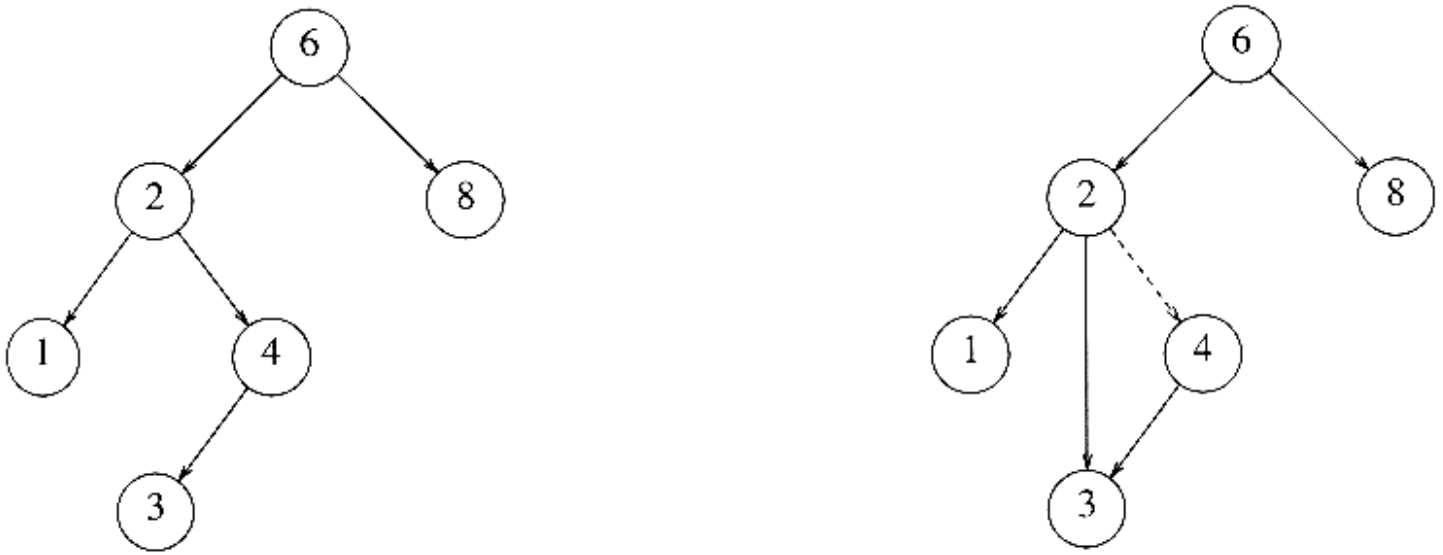


Figure 4.24 Deletion of a node (4) with one child, before and after

delete

(3) the node has 2 children

- replace the key of that node with the minimum element at the right subtree
- delete the minimum element
  - Has either no child or only right child because if it has a left child, that left child would be smaller and would have been chosen. So invoke case 1 or 2.

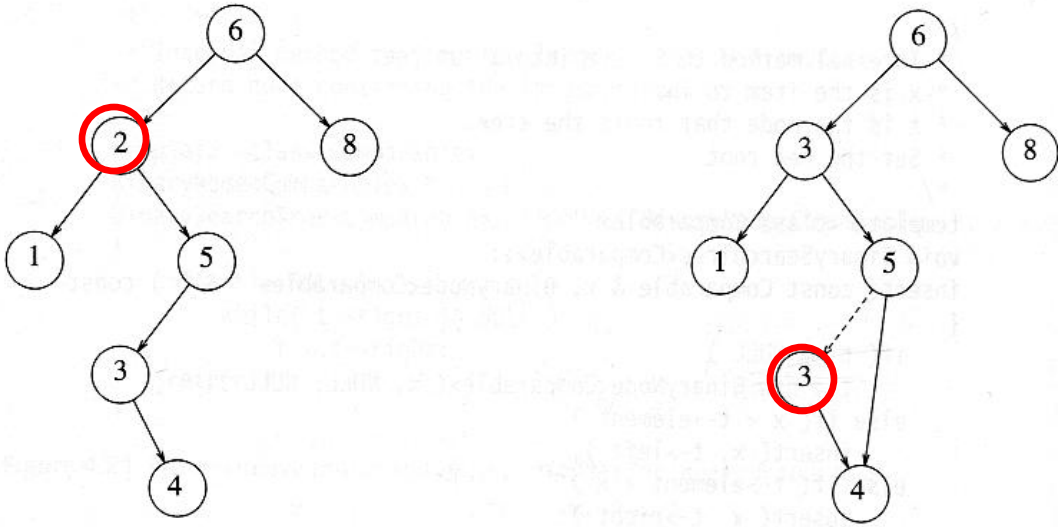


Figure 4.25 Deletion of a node (2) with two children, before and after

- Time complexity = O(height of the tree)